ing. In short, teacher intervention is an act of observation (might the researcher-observer be so bold as to say observing|intervening?). In this case the researcher-observer brings forth the teacher-observer that brings forth the learners-observers as having posed the problem: Can we write all requirements with an identical denominator? Acting from observations, the teacher offers a simple intervention to the group:

Teacher: Is that half blue?
Brock: This is only 14!
Ria: We need 20 tiles.
Sharla: Because the common denominator is 20?
Teacher: What were you going to say, Ria?
Ria: You need 20 tiles because the denominator, the common denominator is 20.
Brock: Yeah. It doesn't equal 20.

« 10 » The result is a flurry of dialogue and, what we observe to be a shift in the posing|solving activity that orients the group's activity. The group begins to fixate on the need for twenty tiles, and explores ways in which they can satisfy the new, posed problem: How can we meet the requirements by using 20 tiles? Caught in the fundamental circularity, the teacher's problem-posing (and subsequent interactions informed through this problem-posing), becomes energy-rich matter (Foerster 1981) that only results in redirection towards curriculum outcomes when that intervention is a trigger for further student problem-posing.

« 11 » When we forefront the teacher as observer, we observe the teacher to be operating informed by a posing|solving of the the posing|solving activity of learners. This problem-posing, which Varela has called "the greatest ability of living cognition" is a part of a circularity of observation involved in the complex ecology of the mathematics classroom (Varela, Thompson & Rosch 1991: 145). It is this attention to the teacher-observer that we wish to add to the contributions of Proulx and Maheux. Namely, that posing|solving is not only a process through which learners bring forth meaning, it is the fundamental work of the teacher.

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Received: 10 October 2017
Accepted: 17 October 2017

Francisco Varela's Four Key Points of Enaction Applied to Working on Mathematical Problems

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> Upshot • After a description of Varela's four key points to a science of inter-being: embodiment, emergence, intersubjectivity and circulation, three questions are asked and briefly explored: Are these key points illustrated in the target article? What is a problem? And what could classrooms look like where knowing is doing?

« 1 » In the conclusion of their target article, Jérôme Proulx and Jean-François Maheux point out that they are taking "inspiration from Varela's work [… making] our ideas and conceptualizations possible" (§37), triggering distinctions they make in their own research processes. Since we cannot interact with Francisco Varela directly we are indeed left with our own inter-actions related to reading his words and then laying down our own path in walking, in walking laying down a path. In 1996, I attended a conference entitled "The psychology of awakening: An international conference on Buddhism, science and psychotherapy," where Varela was one of the keynote speakers. I had started reading Varela's books and papers in 1995, bringing forth, with others, interpretations of his thoughts in the teaching and learning of mathematics and our research.

« 2 » Varela, a neuroscientist, had been asked by a publisher to write a book where the recent advances in neuroscience could be presented in such a way that they could be read on the Paris metro (Varela's lab was in Paris). This trigger led to the articulation in the lecture of four key points, the first three of which would be accepted then by his neuroscientific colleagues and the fourth, given the label emptiness, would not, but he believed that they were close to having scientific evidence for that point too. In the book of the conference, the fourth one was developed and given a different label, circulation, which, with the other three, he believed to be "precise, fundamental insights which rest upon about fifty years of good research, and which I take to be established results" (Varela 1999b: 71).

« 3 » So, are Varela's key points illustrated in the article? What follows are the four key points and a short reflection on how I see each of these fitting with the authors' target article and possible developments:

Key Point 1: Embodiment
The mind is not in the head: Cognition is enactively embodied co-determination of inner/outer

« 4 » So, there is no separation between inner and outer, the world is part of us and we are part of the world simultaneously. We learn by physically exploring,

solving problems as we go without necessarily engaging the brain and the frontal cortex in the head, which only kicks in when there is a problem that the embodied mind in the environment cannot solve immediately through acting. As Andy Clark says, "Minds make motions, and they make them fast" (Clark 1997: 1), the tools that we use, like Heidegger's hammer, become ready-to-hand, able to be used immediately without reflective thought (Heidegger 2010: 69). What happens when the tool does not work as expected? When it is too heavy, for instance, as Heidegger suggests? At that point there is a need for some theoretical scrutiny. As a researcher, observing students individually doing questions given them to solve, it is possible to see that children are using different methods (illustrated in Box 1). However, what would it be like to create a space where the children could extend their awarenesses through interaction? Within Lev Vygotsky's Zone of Proximal Development (ZPD), individuals are able to do more than they thought possible through interaction with peers. The children could become flexible across a number of different methods.

Key Point 2: Emergence
The mind neither exists nor does it not exist: Cognition is enactively emergent co-determination of neural elements (local) and cognitive subject (global)

« 5 » Varela's example of this is himself. He is made up of lots of local elements and yet, globally, he can be recognised as Francisco. And, moreover, without its being seen as magic, emergence implies that it works the other way, Francisco must also be able to influence his parts. Maturana and Varela's work was with living beings with a biological basis of being, but consider a classroom culture. The local parts are the individual children, with their individual knowings. In interaction, there is a class dynamic, where cognition is distributed (Salomon 1993) and we are not getting to a right answer but exploring a territory that is richer for our interconnections. The observer is implicated in the action even when they believe they are separate. In the article, there are examples, including the (interesting to the authors!) example of solving a quadratic expression for its zeroes, that could be read emergently as students using locally many different aspects of their knowledge in solving a global problem (§§10–16).

Key Point 3: Intersubjectivity
This mind is that mind: Cognition is generatively enactive co-determination of Me-Other

« 6 » We are born being able to read the emotions of others and so the image of a classroom where children are concentrating, sitting at individual desks does not feel like they are using intersubjectivity. They are told they need to concentrate and memorize solutions when the natural way of learning is through interacting with the world and other people in it. People grow up being able to read the non-verbal signs and gestures of others, e.g., to know when another is teasing or playing rather than being serious. I do not find this applicable to the article and Varela does talk about it as a "not well charted" area yet in 1999. However, since then, empathy has been a focus of research, see the work of Evan Thompson (2001) and Dan Zahavi (2010).

Key point 4: Circulation
Consciousness is a public affair: Consciousness is ontologically complex co-determination of first- and third- person descriptions

« 7 » Varela was a Buddhist, practising meditation, and a neuroscientist focusing on methods that braided standard scientific methods with first-person accounts in a neurophenomenology. For instance, in his work with epileptics, Varela saw that there were changes in the brain's activity before an incident. His PhD student at the time, Claire Petitmengin, worked with the epileptics in phenomenological-style interviews gaining access to the detail of what happened before an incident so that, with this support, the epileptics came to recognize those changes and find ways to avoid the incident (Petitmengin 2006: 232). In the target article, it seems to me that the authors are talking about students being problem-posers rather than memorising pathways in a heuristic. The students' written methods feel like first-person accounts being accepted in their variety rather than having to conform to a particular method. The teacher is learning their students' mathematics rather than presenting their own. How could this idea be extended to its being the students' classroom and how might first-person or second-person accounts, through an interview, support the research?

« 8 » What is a problem in the context of this article? There is certainly a large literature in mathematics education research about problems, problem-solving and problem-posing, and, talking with researchers and teachers from other countries, it becomes apparent that what we, as academics, are reading through an article or paper varies dependent upon our histories and cultures, like the children's solutions in the paper. To some, for instance, problems mean "word problems," to others, "real-life problems" and so on. How could it be otherwise? (Key point 1). The multiple perspectives illustrated by different methods of solution from the students I was comfortable with, but in what ways was what was offered to the students a problem? The authors are Canadian Francophones, and I do not know enough about their histories of engagement with the culture of learning they have grown up with. An article, "Problem solving in France" (Artigue & Houdement 2007) uses terms such as dialectical and open problems that triggered an awareness of the potential of links to the French problem-solving traditions of Guy Brousseau and Yves Chevellard in the authors' histories that are being used out of that history. I became aware that I would not have applied "dialectical" to problem-solving, given Key Point 3, intersubjectivity. Out of my history, classroom spaces are where children are working together triggering and being triggered simultaneously. No matter how often I work in international collaborative groups, I need to be reminded, as here, that there is a lot of work to do in asking questions about the meanings of words that we take for granted in our own cultures.

« 9 » So, what could classrooms look like where knowing is doing? My colleague Alf Coles and I (Coles & Brown 2016: 157) have described a list of aspects of task design for contingent teaching:

- considering at least two contrasting examples (where possible, images) and collecting responses;

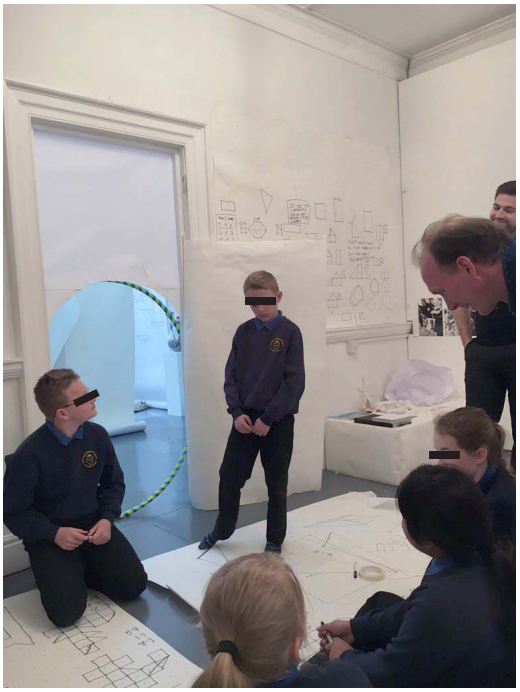

Figure 1 • Entering the exploration space through a hoop.

Figure 2 • Inter-acting with the space.

- asking students to comment on what is the same or different about contrasting examples and/or to pose questions;
- introducing language and notation arising from student distinctions;
- starting with a closed activity (which may involve teaching a new skill);
- having a challenge prepared in case no questions are forthcoming;
- opportunities for the teacher to teach further new skills and for students to practise skills in different contexts;
- opportunities for students to spot patterns, make conjectures and work on proving them, hence involving generalising and algebra."

« 10 » Contingent teaching is used to focus on the teaching being subordinate to what the children are bringing to the space. It is not a heuristic in that the points are not sequenced. The first three points are most directly related to our enactive view of knowing and learning, asking the children to act, immediately coping with their environment (that includes one another) that in turn triggers them to make more distinctions, quite literally to see more. "What do you see?" is a favourite starting point in relation to an image, and discussion of the differences in what is seen between the students leads to questions arising.

« 11 » In Figures 1 and 2 are images from an exploration space that my colleague Coles was involved in, where young children literally enter through a hoop (Figure 1) and can continue with their explorations, being able to be triggered by everything that has been and is being done, anywhere on the papered floors and walls of the space. For researchers, given a particular research question, collecting the children's different methods could be similar to Boxes 1 and 2 although the key point of intersubjectivity is not interested in each separate method, perhaps, but the ways these inter-act when this mind is that mind, reminiscent of Vygotsky's idea of our being able to do more with peers than alone. Key point 4, circulation, would also suggest that for a neurophenomenological study, first- and third-person (or second-person) accounts would be collected.

Acknowledgements

I would like to thank my colleague Alf Coles, who appears in Figures 1 and 2, for the photographs, and the children at St Andrew's Primary school in Bath, UK.

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Received: 9 October 2017
Accepted: 17 October 2017